DeFT: A Conceptual Framework For Considering Learning with Multiple Representations

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Abstract

Multiple (external) representations can provide unique benefits when people are learning complex new ideas. Unfortunately, many studies have shown this promise is not always achieved. The DeFT (Design, Functions, Tasks) framework for learning with multiple representations integrates research on learning, the cognitive science of representations and constructivist theories of education. It proposes that the effectiveness of multiple representations can best be understood by considering three fundamental aspects of learning: the design parameters that are unique to learning with multiple representations; the functions that multiple representations serve in supporting learning and the cognitive tasks that must be undertaken by a learner interacting with multiple representations. The utility of this framework is proposed to be in identifying a broad range of factors that influence learning, reconciling inconsistent experimental findings, determining under-explored aspects of learning with multiple representations and pointing forward to potential design heuristics for learning with multiple representations.

1. Introduction

Research on learning with representations has shown that when learners can interact with an appropriate representation their performance is enhanced. Recently, attention has been focused on learning with more than one representation, seemingly predicated on the notion 'that two representations are better than one'. Yet, as research on learning with multiple external representations (MERs) has matured, it is increasingly recognised that the issue is not whether MERs are effective but concerns the circumstances that influence the effectiveness of MERs (see Goldman, 2003).

The most common approach to considering the effectiveness of representations emphasises the sensory channel and/or the modality of the representations (i.e. either auditory/visual, or textual /pictorial). Two theories that are particularly associated with this approach are the Cognitive Theory of Multimedia Learning (e.g. Mayer, 1997, 2001) and Cognitive Load theory (*e.g.* Sweller, van Merrienboer, & Paas, 1998). They share a focus on the nature of working memory (and its relation to long term memory) with its multiple, modality-specific limited capacity subsystems. Presenting information in multiple modalities is seen as advantageous to learners who actively process such information. Schnotz (2001; 2002; Schnotz & Bannert, 2003) focuses not on pictures and text *per se*, but on depictive (iconic) and descriptive (symbolic) representations. In this approach, mapping happens at the level of mental model construction and what results is not an integrated representation but complementary representations that can communicate with one another.

The purpose of this paper is to present an alternative approach addressing different aspects of learning with representations. Instead of focusing on the form of the representational system, it suggests that there are a number of additional design factors that should be considered. Given its wider scope it is premature to advance design principles. So, instead of proposing predictive guidance, it aims to suggest a complementary set of factors that should guide research into design of effective multi-representational software. Thus, this paper serves as a review of research on MERs, an argument about the importance of acknowledging a wide range of factors that influence learning with MERs and some proposed applications of this approach.

A Framework For Learning with Multiple Representations

The DeFT (Design, Functions, Tasks) framework suggests that many dimensions combine to influence whether an individual learner will be able to benefit from learning with a particular combination of representations. The dimensions considered in DeFT are the ***de***sign parameters that are unique to learning with more than one representation, the different pedagogical ***f***unctions that MERs can play, and the cognitive ***t***asks that must be undertaken by a learner when interacting with MERs. This framework has been developed over the past five years by reviewing a broad range of current research from a variety of perspectives (e.g. cognitive psychology/science, education, artificial intelligence, educational and curriculum studies), addressing methodologies such as case studies, experiments and computational modelling, and from empirical work conducted in domains as a varied as mathematics, physics, biology, and alchemy.

Understanding the role played by MERs first requires understanding how external representations influence learning. Consequently, this paper begins by briefly describing the design factors that DeFT addresses before turning to the advantages of employing the right representation (the functions aspect of the framework) and cognitive tasks associated with learning with a single external representation (tasks). Then these same issues are considered when learning with multiple representations, considering first the advantages of MERs (functions) and then the unique demands upon the learner that MERs presents (the tasks aspect). The final sections review how DeFT might be applied to increase understanding of the impact of different designs on learning with MERs.

2. Design Parameters in DeFT

There are a number of ways to design multi-representational systems that influence the processes and outcomes of learning. Systems differ in their content, in the target users of the system and in the teaching strategies they employ. Often there are specific reasons to use a particular representation. An external representation consists of (1) the represented world, (2) the representing world, (3) what aspects of the represented world are being represented, (4) what aspects of the representing world are doing the modelling and (5) the correspondence between the two worlds (Palmer, 1977). So when considering the effectiveness of a representation both the information provided in the representation (represented world) and the way it is presented (representing world) must be considered. Consequently, designing effective representations is substantial endeavour in its own right. However, there are a set of design dimensions that uniquely apply to multi-representational systems and it is these that are reviewed here: (a) the number of representations employed; (b) the way that information is distributed over the representations; (c) the form of the representational system; (d) the sequence of representations; and (e) support for translation between representations.

Number. By definition, multi-representational systems employ at least two representations. Commonly many more representations are available in a system either simultaneously or at some point during a learners' interactions with it. However, an excessive number of representations rarely helps learning.

Information. Multi-representational systems can allow flexibility in the way that information is distributed between the representations. Consequently, this impinges on the complexity of the information in each representation and the redundancy of information between representations. At one extreme, each representation can convey completely different content (refer to different represented worlds). In this case, there is no redundancy across representations. Distributing information in this way may simplify each individual representation but at the cost of requiring additional representations. Learners may then be required to integrate information from multiple sources. Systems can also be partially redundant, so that some of the information is constant across (some of) the representations.. Finally, each representation could be designed to express the same information and so the only difference between the representations is in their computational properties (representing worlds). In this case, there is full informational redundancy across the system but often each representation is more complex.

Form. A typical multi-media system can display pictures, text, animations, sound, equations, and graphs, often simultaneously. A key question is whether it should. Much research has focussed on heterogeneous systems - ones that combine text and graphics (Barwise & Etchemendy 1992; Mayer, 1997; Schnotz 2002) or multi sensory systems such as simultaneous written text or pictures presented with spoken narration (Kalyuga, 2000; TindallFord, Chandler, & Sweller, 1997). However, form can refer to many different aspects of representations. For example, dynamic and static multi-representational systems have different properties, as do 2d, 3d or mixed 2d and 3d systems. Consequently, much is still unknown about how the form of a representational system influences learning. To understand learning with MERs it is not sufficient to consider each type of representation in isolation - representations interact with one another in a form of "representational chemistry". As a result, there is a potentially vast space to explore.

Sequence. If not all representations are draw upon simultaneously, a number of further issues arise. The first issue is the sequence in which the representations should be presented or constructed. Even if a sequence has been predetermined, the learner or the system still needs decide at what point to add a new representation or switch between the representations.

Translation. Computerised environments have made possible a wide variety of ways to indicate to learners the relation between representations. Two dimensions have received some attention. Firstly, how active a role the environment plays in supporting learners. Secondly, whether support is provided at the syntactic level or the semantic level (also called surface or deep levels or representation or domain levels (Seufert & Brüncken, 2004).

In the next sections, firstly the pedagogical functions and cognitive tasks associated with learning with one representation and secondly with multiple representations are considered. Then, these design parameters are reconsidered to examine if considering learning in this way helps designers with these complex decisions.

3. Learning with an External Representation

3.1. The Functions of an Appropriate Representation

There is abundant evidence showing the advantages that external representations play in supporting learning (e.g. White, 1993; Winn, 1987). Much research has shown that matching the type of representation to the learning demands of the situation can significantly improve performance and understanding. Scaife & Rogers (1996) proposed that external representations differ in their advantages for learning by varying the extent to which they support computational offloading, re-representation or graphical constraining. These advantages of different representations means that combinations of representations can play a number of functions in supporting learning (discussed in detail in section 5)

Computational offloading is the extent to which different external representations reduce the amount of cognitive effort required to solve equivalent problems. Larkin & Simon (1987) argue that representations that are informationally equivalent still differ in their computational properties. For example, diagrams can exploit perceptual processes by grouping together relevant information so that make processes such as search and recognition easier. Alternatively representations such as tables tend to make information explicit, emphasise empty cells that directs attention to unexplored alternatives, and allow quicker and more accurate readoff (e.g. Meyer, Shinar, & Leiser, 1997).

Re-representation refers to the way that alternative external representations, which have the same abstract structure, differentially influence problem solving. Zhang & Norman (1994) showed that problem solving with isomorphic versions of the Towers of Hanoi was enhanced when representations externalised more information. By utilizing external perceptual processes rather than cognitive operations, graphical representations will often be more effective.

Graphical constraining describes the limits on the range of inferences that can be made about the represented concept. Stenning & Oberlander (1995) argue that text permits expression of ambiguity in a way that graphics cannot easily accommodate. It is this lack of expressiveness that makes diagrams more effective for solving determinate problems. Schnotz (2001; 2002) makes a similar point when emphasising the distinction between descriptive and depictive representations. Descriptive representations are symbolic in nature whereas depictive representations are iconic. Thus, depictive representations are most useful to provide concrete information about specific instantiations of the object of interest and are often efficient as specific information can just be readoff. Alternatively descriptive representations can more easily express abstract information and more general negations and disjunctions.

3.2. Cognitive tasks involved in Learning with an External representations

Unfortunately, the benefits of appropriate representations do not come for free. Learners are faced with complex learning tasks when they are first presented with a novel representation. They must come to understand how it encodes information and how it relates to the domain it represents. In addition to these required tasks, learners may be need to select an appropriate representation or to construct one for themselves, which can provide advantages but also new cognitive tasks. The following sections describes research addressing each of these cognitive tasks and the problems learners can face in mastering them. The level of analysis is above the granularity of fine-grained perceptual analysis (e.g. Legge, Gu, & Luebker, 1989). Finally, it should be noted that although these cognitive tasks are presented in sequence, it not meant to imply that learners would approach the task of understanding a new representation in this same order.

3.2.1. Learners should understand the form of representation

Learners must know how a representation encodes and presents information (the ‘format’). In the case of a graph, the format would be attributes such as lines, labels, and axes. They must also learn what the ‘operators’ are for a given representation. For a graph, operators to be learnt include how to find the gradients of lines, maxima and minima, and intercepts.

A number of studies have shown how difficult this is (e.g. Friel, Curcio, & Bright, 2001). Preece (1993) reports that 14-15 year old children experienced difficulty in applying and understanding the format and operators of graphs. For example, some pupils have trouble with reading and plotting points, they interpreted intervals as points, and confused gradients with maxima and minima. Petre & Green (1993) describes some similar effects when adults are learning to understand a visual interface. In observing differences between novices and experts, they showed that novices lack proficiency in secondary notation (i.e. perceptual cues that are not described by the formal semantics of a representation). Novices find navigation of graphical representations difficult as they don’t have the required reading and search strategies and, in contrast to expert performance, they tend not to match strategies to the available representations.

Additionally, the operators of one representation are often used inappropriately to interpret a different representation – for example, when graphs are interpreted iconically, learners inappropriately use the operators for pictures (e.g. Leinhardt, Zaslavsky, & Stein, 1990). When learners are given a velocity-time graph of a cyclist travelling over a hill, they should select a U shaped graph, yet many show a preference for graphs with a hill shaped curve (e.g. Kaput, 1989). Elby (2000) proposes that in many of these cases learners tend to rely on an intuitive knowledge element - what-you-see-is-what-you-get and that this is cued by the most compelling visual attribute of a representation (e.g. straight lines mean constancy, hill shape means hill). Learning to interpret a representation can involve learning to ignore this intuition.

3.2.2. Learners should understand the relation between the representation and the domain

Interpretation of representations is an inherently contextualised activity (e.g. Roth & Bowen, 2001) as learners must also come to understand the relation between the representation and the domain that it represents. This task will be particularly difficult for learning, as opposed to problem solving or professional practise, as this understanding must be forged upon incomplete domain knowledge. Learners need to determine which operators to apply to a representation to retrieve the relevant domain information. For example, when attempting to read the velocity of an object from a distance-time graph, children often examine the height of line, rather than its gradient (Leinhardt *et al*, 1990). Brna (1996) shows even fairly competent programmers who had received information about the elements of a new (visual programming) representation failed to clearly map the format of the new representation onto their existing domain knowledge. These problems do not only arise with abstract representations. Boulton-Lewis & Halford (1990) point out that even concrete representation such as Dienes blocks and fingers still need to be mapped to domain knowledge. Processing loads may be too high for children to obtain the anticipated benefits of such apparently simple representations.

3.2.3. Learners may need to understand how to select an appropriate representation

In some situations learners select a representation that they find most appropriate, and so they may have to consider such aspects of the situation as the representation and task characteristics as well as individual preferences. There is evidence in certain situations that learners can select effective representations. Zacks & Tversky (1999) found that people were successful at choosing bar graphs to represent discrete comparisons between data points and line graphs to depict trends. Novick, Hurley & Francis (1999) found that students were able to choose which of hierarchical, matrix or network representations was most appropriate to represent the structure of a story problem, presumably based upon abstract schematic knowledge about when best to apply particular representations. However, there may well be individual differences in insight into effective representation for tasks (Roberts, Gilmore, & Wood, 1997). Cox (1996) found that learners without good insight into the problem tended to “thrash about” – choosing representations without moving themselves nearer to a solution. Again, it is likely that selecting appropriate representations will be more difficult for novices than experts as they can lack understanding of the deep nature of the tasks they are trying to solve (Chi, Feltovich, & Glaser, 1981). Indeed one characteristic of expertise is the knowledge of what representations are appropriate for what tasks (e.g. Kozma & Russell, 1997). One key unsolved issue is how explicitly these skills should be taught; with some researchers argue that teaching is crucial (McKendree, Small, Stenning, & Conlon, 2002).

3.2.4. Learners may need to understand construct an appropriate representation

In many situations learners may be required to construct a representation rather than interpret a presented representation. Learners often construct their representations inaccurately (e.g. Cox 1996). However, Cox also found that learners could sometimes draw the correct inference even if they form incorrect representations. There is evidence that creating representations can lead to a better understanding of the situation. Grossen & Carnine (1990) found that children learned to solve logic problems more effectively if they drew their responses to problems rather than selected a pre-drawn diagram. This may in part be due to the support they were provided with during the construction process. Van Meter (2001) found that drawing was most helpful in learning from science texts when students were prompted with guidance questions whilst creating diagrams.

There is unlikely to be a simple relationship between interpretation and construction of a representation. Cox (1997) gave students reasoning problems which could be solved with Euler diagrams. He found six students made errors in constructing representations but not in interpreting them. However, four students made errors in interpreting diagrams but not in constructing them. If learners are free to construct their preferred representation then the process of interpreting constructed representations should be easier. Presumably learners will be more familiar with the form of the representation and how its relate to the domain.

4. Learning With More than One External Representation

Early research on learning with MERs concentrated on the ways that presenting pictures alongside text could improve readers’ memory for text comprehension (e.g. Levin, Anglin, & Carney, 1987). In the last two decades, the explosive increase in multi-media learning environments have widened the debate to include an extensive variety of multi-representational systems formed from combinations of representations such as diagrams, equations, tables, text, graphs, animations, sound, video, and dynamic simulations. Furthermore, a number of influential educational theories discuss the importance of MERs. For example, Dienes (1973) argues that perceptual variability (the same concepts represented in varying ways) provides learners with the opportunity to build abstractions about mathematical concepts. In cognitive flexibility theory, the ability to construct and switch between multiple perspectives of a domain is fundamental to successful learning (Spiro & Jehng, 1990).

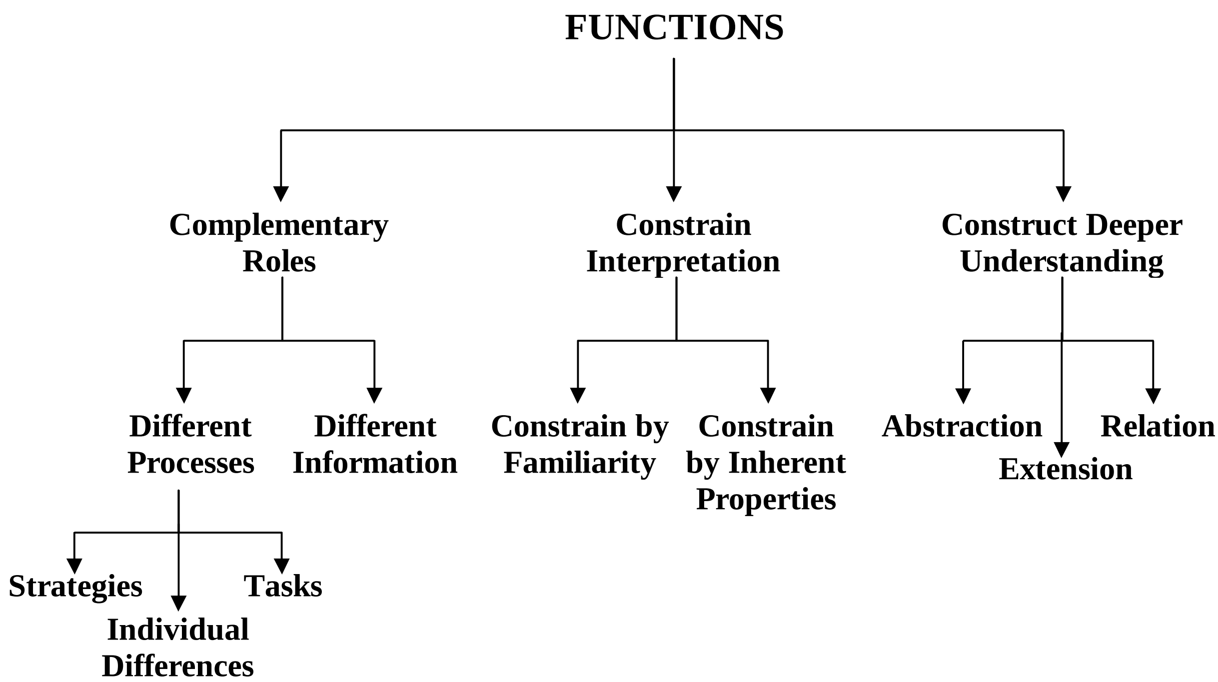
However, research on the benefits of providing learners with more than one representation has produced mixed results. For example, a number of studies have found that learners benefit from either constructing or being presented with MERs (e.g. Cox & Brna, 1995; Mayer & Sims, 1994; Tabachneck, Koedinger, & Nathan, 1994). Unfortunately, just as many studies have shown that learners can fail to benefit from these proposed advantages of MERs (e.g. Chandler & Sweller, 1992; de Jong *et al*, 1998; Scanlon, 1998; Tabachneck-Schijf & Simon, 1998). Typically, these studies have shown learners tend to treat representations in isolation and find it difficult to integrate information from more than one source.

In the next sections, the benefits that MERs can bring to learning situations are reviewed. The Functions aspects of the DeFT framework proposes that there are many advantages of MERs and that these should be clearly identified as they often have different design implications. These functions of MERs are only possible if learners master the cognitive tasks associated with their use. Learning to use MERs requires learners to understand each individual representation. This is a complex process in its own right (see 3.2). But, in addition, when interacting with MERs, learners must often understand the relationship between representations and there is evidence that this is particular difficult. Hence, this cognitive task is considered at some length.

5. Functions

Ainsworth (1999) suggest there are three key functions of MERs: to complement, constrain and construct (see Figure 1).

Figure 1. A Functional Taxonomy of Multiple Representations



5.1. Complementary Functions

When MERs complement each other they do so because they differ either in the processes each supports or the information each contains. By combining representations that differ in these ways, it is hoped that learners will benefit from the advantages of each of the individual representations.

5.1.1. Complementary processes

Representations that theoretically contain the same information still differ in their advantages for learning in certain situations due to the extent to which they support computational offloading, re-representation or graphical constraining (Scaife & Roger, 1996). This function of MERs presumes that by providing MERs allows complementary processes to be supported. This can be advantageous for a number of reasons.

*Individual Differences***.** Theorists following a learning styles approach argue that if learners are presented with a choice of representations, they can choose to work with the representations that best suits their needs (e.g. Dunn & Dunn, 1993). There is limited evidence that this can improve learning. For example, Plass, Chun, Mayer, & Leutner (1998) found that students comprehended a story in a second language better when they had the opportunity to receive their preferred mode of annotation (visual/verbal/both). However, the assumption that people assessed or self-reported as visualisers will necessarily do better with visual representations is not always warranted (Dekeyser, 2001; Klein, 2003; Roberts *et al*., 1997). Coffield *et al* (2004) provide a comprehensive review of the controversy surrounding learning styles. Alternative accounts of the individual differences effect emphasise differing expertise with the subject studied or with alternative forms of representation. For example, Stenning, Cox & Oberlander (1995) gave learners either a graphical or non-graphical course on logic. They found that high performing students benefited for the graphical treatment but lower performing students performed better when given traditional textual instructions. ChanLin (2001) found novices learning physics benefited from the use of still graphics rather than text but found no differences between formats for experienced students.

Task. Performance is most likely to be facilitated when the structure of information required by the problem matches the form provided by the representational notation (Gilmore & Greene, 1984). Learners given MERs can benefit from choosing the best representation for the current task. Examples of this effect can be seen in Bibby & Payne (1993) who examined how informationally equivalent tables, procedures and diagrams supported acquisition of device knowledge. Looking at performance on a simple control panel device, they found cross over effects. Subjects given tables and diagrams identified faulty components faster. However, those given procedures were faster at deciding which switches were mispositioned. Tapiero (2001) found that when subjects were given textual descriptions of a city, they performed spatial judgement tasks more accurately than those given a map of the city. However, when presented with a transfer task of a map of a modified city, the reverse was true - map subjects performed better than text subjects.

*Strategy***.** Different forms of representation can encourage learners to use more or less effective strategies (e.g. Ainsworth & Loizou, 2003). MERs encourage learners to try more than one strategy to solve a problem. Tabachneck *et al*, (1994) examined the representations that learners created to solve algebra word problems and found each representation was associated with different strategies. The use of MERs and hence multiple strategies was about twice as effective as any strategy used alone. As each strategy had inherent weaknesses, switching between strategies made problem solving more successful by compensating for this. Cox (1996) observed a similar effect when students solved analytical reasoning problems. He found that subjects tended to switch between representations at impasses and on difficult problems.

5.1.2. Complementary information

Multiple representations are used to provide complementary information when each representation in the system contains (some) different information. This may occur if a single representation would be very complicated if it presented all the information or if the information is on radically different scales.

A typical example of this function is provided by "MOLE", a learning environment for modal logic (Oliver, 1998). It contains two representations: one a node and link description of the relation between different modal worlds and the second is a grid of polygons that illustrate the content of each world individually. Oliver found that this allowed learners to concentrate on different aspects of the task, making the learning goals more achievable.

5.2. Constraining Functions

A second advantage of using MERs is that certain combinations of representations can help learning when one representation constrains interpretation of a second representation. This can be achieved in two ways. Firstly, learners' familiarity with one representation can constrain interpretation of a less familiar one. For example, concrete animations are often employed in simulations alongside complex and unfamiliar representations such as graphs. Secondly, these constraints can be achieved by taking advantages of inherent properties of representations. Graphical representations are generally more specific than textual representations (Stenning & Oberlander, 1995). If someone is provided with the natural language expression 'the knife is beside the fork', there is inherent ambiguity about which side of the knife the fork has been placed. This is not possible when representing the same world pictorially, since the fork must be shown as either to the left or to the right of the knife (e.g. Ehrlich & Johnson-Laird, 1982). So, when these two representations are presented together, interpretation of the first (ambiguous) representation may be constrained by the second (specific) representation. Depictions can perform this same role for descriptions (e.g. Schnotz, 2001).

5.3. Constructing Functions

Multiple representations support the construction of deeper understanding when learners integrate information from MERs to achieve insight it would be difficult to achieve with only a single representation. Furthermore, insight achieved in this way increases the likelihood that it will be transferred to new situations (Branford & Schwartz, 1999).

Abstraction is the process by which learners create mental entities that serve as the basis for new procedures and concepts at a higher level of organization. Learners can construct references across MERs that then expose the underlying structure of the domain represented. Schwartz (1995) showed that the representations that emerge with collaborating peers are more abstract than those created by individuals - abstracted representation may emerge as a consequence of requiring a single representation that could bridge both individuals' representations.

Extension can be considered as a way of extending knowledge that a learner has from a known to an unknown to representation, but without fundamentally reorganizing the nature of that knowledge. For example, learners may know how to interpret a velocity-time graph in order to determine whether a body is accelerating. They can subsequently extend their knowledge of acceleration to such representations as tables, acceleration-time graphs, etc.

Relational understanding is the process by which two representations are associated again without reorganization of knowledge. The goal of teaching relation between representations can sometimes be an end in itself. For example, much emphasis is placed on learning how to construct a graph given an equation (e.g. Dugdale, 1992). On other occasions, it may serve as the basis for abstraction.

It should also be noted that whether representations serve to support extension or relations often depend upon learners' knowledge not system designer's intent. For example, one learner may be familiar with tables and extend their knowledge to graphs (extension), another may already be familiar with both but not have considered their relationship (relation). The differences between these functions of MERs are subtle and all may be present at some stage in the life cycle of encouraging deeper understanding with a multi-representational environment.

5.4. Functions Summary

MERs can play many advantageous roles in learning complex material and these different roles fall into three distinct categories. However, the picture is complicated by the need to acknowledge that MERs can support more than one of these roles simultaneously. For example, Ainsworth, Wood, & O'Malley (1998) found that a combination of table and place-value representations in a primary maths environment provided different information and supported complementary processes, constrained interpretation in two alternative ways and might also have supported abstraction.

6. Cognitive Task: Learners May need to Understand how to Relate Representations

Many of the proposed benefits of MERs result from the integration and co-ordination of more than one source of information and a characteristic of expertise is the ability to integrate different representational formats (e.g. Kozma, Chin, Russell, & Marx, 2000). Kaput (1989, pp. 179-180) proposes that "*the cognitive linking of representations creates a whole that is more than the sum of its parts…it enables us to see complex ideas in a new way and apply them more effectively.*" Unfortunately, a very large number of studies have observed that learners find translating between representations difficult (e.g. Anzai, 1991; Schoenfeld, Smith, & Arcavi, 1993). Learners can fail to notice regularities and discrepancies between representations (Borba, 1994; Dufour-Janvier, *et al* 1987). The impact of failing to relate representations can even completely inhibit learning. Ainsworth *et al* (2002) contrasted children learning estimation with two representations, either mathematical, pictorial or a mixed system of one pictorial and mathematical representation. In one experiment, they showed that whilst pictorial and mathematical representations helped learning, the combination of pictorial with mathematical representations inhibited learning of the task. It was possible to isolate the problem as resulting from relating representations. Each representation in the mixed system was present in either mathematical or pictorial systems where it was used successfully, so it is known that learners could understand the form of representations and their relation to domain, just not how they relate to each other in the mixed case.

Teaching learners to coordinate MERs has also been found to be a far from trivial activity. Yerushalmy (1991) provided students with an intensive three-month course with multi-representational software that taught functions. In total, only 12% of students gave answers that involved both visual and numerical considerations and those who used two representations were just as error prone as those who used a single representation. Resnick & Omanson (1987) gave children mapping instructions about the correspondence between Dienes blocks and written numerals to help them master the symbolic procedures involved in subtraction. They were disappointed by how little children referred to the blocks and found, for the most part, it was not helpful.

Consequently, it is important to understand the factors that influence the difficulty of relating representations. Here the characteristics of the representations and characteristics of the learner are considered.

6.1. Representation Characteristics

A reasonable heuristic for considering how the representational system will impact upon how learners integrate information from multiple sources is to suggest that the more the formats of the representations and the operators that act upon them differ, the more difficult it will be for learners to translate between them. There are a number of dimensions that are likely to maximise these differences between representations including:

*The sensory channel of the representation***.** A common combination of representations is one that combines an auditory with a visual representation. A number of researchers working in the cognitive tradition (e.g. Mayer, 1997; Kalyuga, 2000) propose referential connections between these types of representations are facilitated as combining both auditory and visual stimuli take maximum advantage of short-term memory and so facilitates translation between representations (the Modality Principle). However, Gyselinck, Ehrlich, Cornoldi, de Beni, & Dubois (2000) show that visual-spatial working memory can still be heavily loaded in such situations.

The modality of the representations. A heterogeneous systems is one that contains both a text based representation and a graphical/diagrammatic representation. These representations are known to have very different computational proprieties (e.g. Larkin & Simon, 1987) and some researchers also consider them to be processed separately in the brain (e.g. Mayer, 1997; Tabachneck-Schijf, *et al*, 1997). Consequently, learners may find it difficult to see the relationship between such different forms of representation

The level of abstraction. Peirce (1906) distinguishes between a symbol, which has an arbitrary structure (a description in Schnotz's terminology), and an icon (depiction) that does not. Bruner (1966) adds an extra mode to express knowledge, enactive, to represents events through motor responses. Purchase (1998) further adapts Bruner's scheme by dividing the iconic category in two: concrete-iconic, which has a direct perceptual relationship to the object and abstract-iconic which has a related but non-direct relation. Blackwell & Engelhardt (1998) identify eight schemes that consider level of abstraction. There seems little agreement about the granularity of the dimension but its importance is widely recognised.

The specificity of representations. Specificity determines the extent to which a representation permits expression of abstraction (Stenning & Oberlander, 1995). This is proposed to a representation's cognitive computational properties. Learners will interpret and act upon representations of different levels of specificity in unlike ways, so they may find integrating information across representations that differ in specificity more difficult.

The type of representation (e.g. histogram, equation, table, line-graph, narrative text, picture). There are many schemes proposed for categorising representations into different types (e.g. Cox, 1996; Lohse, Biolsi, Walker, & Rueler, 1994). For example, Lohse *et al*, identified eleven major clusters: graphs, numerical and graphical tables, time charts, cartograms, icons, pictures, networks, structure diagrams, process diagrams and map clusters. These taxonomies have been created by a variety of methods (e.g. intuition, analysis of domain properties and card sort techniques with subjects) and although there is some overlap between the taxonomies, no one classification is universally accepted. They differ in the domains addressed, the granularity with which representations are described and the task for which they were created. However, if learners consider that two representations are of alternative types, they may well manipulate and interpret them differently.

Integrated presentations of representations. When presenting textual and graphical representations learners find it easy to understand physically integrated material rather than separately presented material (Chandler & Sweller, 1992).

Whether representations are static or dynamic. Dynamic representations such as animations, dynamic graphs and spoken text require different operators to interpret them and have different formats than their static equivalents. This can be shown by the way that learners draw different inferences from pictures and animations (e.g. Jones, 1998; Lowe, 1999; Pedone, Hummel & Holyoak, 2001). Thus, representational systems that combine static and dynamic representations may be particularly complex. Furthermore, different types of dynamic representations also have different format and operators (Ainsworth & Van Labeke, 2004).

Dimensionality. With the mounting availability of virtual reality and other visualisation tools, learners are increasingly being placed in situations where they must integrate information from both 2D and 3D representations. There is evidence that learners can fail to build these links easily (Moher, Johnson, Ohlsson, & Gillingham, 1999).

6.2. Individual Characteristics

How well individuals cope with the relating different representations is likely to depend upon a number of learner characteristics. Probable candidates include familiarity with the representations and domain, age and cognitive style.

Representational Familiarity. If learners are already familiar with the representations, then they should understand (to some degree) the format and operators of representation and the relation between the representations and the domain. The lower the learning demands are on other parts of the task, the more resource for translating between representations should be available. Furthermore, if learners are less likely to misinterpret the representations, this should enhance the possibility of recognising the similarity between them.

Domain familiarity. Generally, it has been shown that novices tend to characterise problem representations by their surface features, not their deep structure (e.g. Chi *et al*., 1981). Therefore, as learners generally lack expertise either in the domain or the representations they are using, they are likely to be hampered in recognising deep structural relations between representations due to their surface dissimilarity. This lack of domain knowledge interferes with their ability to transfer knowledge across representations appropriately (e.g. Stern, Aprea & Ebner, 2003).

Age. A learner's age may also affect their abilities to translate between representations. Often children's performance can be seen as characteristic of novices in a domain. Nevertheless, there are likely to be developmental factors that affect integration of MERs. Moore & Scevak (1997) found developmental differences in children use of text and accompanying visual aids. There was explicit linking of text and visual aid information for older students that was not as evident in the younger students. A number of researchers have proposed that information-processing capacity increases with age. Candidates include short-term memory span (e.g. Case, 1985), processing speed (e.g. Vernon, 1987) and central computing space (Pascual-Leone, 1970). Of particular relevance is Halford's description of dimensionality, which is defined as number of independent items of information that must be processed in parallel (Halford, 1993). He proposed that it is not until children reach eleven years of age that they can process four-dimensional structures. If MERs exceed this capacity then children would need to re-represent the problem for example by chunking. This suggests that younger children would require considerable experience with the representations in order to relate them successfully.

Individual differences. There has been much research relating both personality and cognitive factors to learning with external representations (see 5.1.1). There is less research into aptitude-treatment interactions and MERs. An exception is that of Oberlander, Cox, Monaghan, Stenning, & Tobin (1996). They suggest that a distinguishing characteristic of people who were classified as diagrammatic reasoners was their ability to translate information across representations more successfully.

6.3. Representation and Individual Characteristics

These two levels come together in the way that individual factors and representation factors influence the strategies that learners use. Using MERs can encourage learners to use different strategies (e.g., Tabachneck, *et al*, 1994; Watson, Campbell, & Collis, 1993). This is often advantageous as by switching between representations learners can compensate for weaknesses in the strategy or representation. However, if learners are attempting to relate different representations then this may provide a source of difficulty. Ainsworth *et al* (2002) hypothesised that one of the reasons why learners did not integrate information across in pictorial and mathematical representations is that the pictorial representation encouraged the development of a perceptual strategy and the mathematical one encouraged learners to generate a rule based upon symbol manipulation.

7. Applying DeFT to Explore Different Designs

In section 2, the (implicit or explicit) design parameters of multi-representational software were identified. In this section, I return to these parameters to explore if considering the functions and cognitive tasks of learning with MERs can help someone make these design decisions. It is fair to say that as this stage DeFT raises far more questions than it answers. However, it may still be useful if it helps to identify those aspects of multi-representational design that are under-explored. DeFT may also encourage researchers to describe in more detail the design parameters of their systems and the pedagogical functions they intend to their systems to play. Although, research might initially be considered as addressing one question (e.g. simultaneous versus sequential presentation), other researchers interested in alternative questions (e.g. redundancy) are often unable to draw conclusions about their interests if this aspect of the system is not described.

7.1. Number

Given the research reviewed on the cognitive tasks associated with adding representations to a system, it seems wise to use the minimum number of representations consistent with the pedagogical function of the system. In many cases it may not be appropriate to use MERs at all, since one representation may be sufficient and will minimise the split attention affect. However, there are many circumstances where MERs are appropriate. The decision about the number of representations often depends upon the informational (section 7.2) and computational (section 7.3) properties of the desired representational system.

7.2. Information

Information can be distributed in multiple ways over MERS which may simplify individual representations and impact upon the redundancy of the representational system. Consequently there may be a way of distributing information that best supports learning (for a particular task and a particular type of learner). One possibility is that it is easier to learn complex ideas when each part is represented separately in a simpler representation. Alternatively, it easier to learn from complex representation(s) as all the information is presented together.

Kalyuga, Chandler, & Sweller (1998) report a number of studies that showed that less experienced learners benefit from redundant text but for those with more experience the same text interfered with performance with diagram. Learners who can benefit from the diagram in isolation do not need text and so eliminating it reduces cognitive load. This suggests that redundancy should be reduced as expertise grows. However, Ainsworth, Bibby, & Wood (1997) gave students two representations to describe their performance on computational estimation tasks – these were either non-redundant where each representation provides one dimension of information each or completely redundant with both representations displaying two dimensions of information. They found that learners given non-redundant representations understood aspects of estimation accuracy faster than those given fully redundant representations. Examining the apparent contradiction between these experiments, it seems likely that the difference rests of the functions of the MERs. The text in Kalyuga *et al*'s studies seems to be used by novices to constrain interpretation of an unfamiliar diagram, whereas in Ainsworth *et al*'s study, the representations are used to complement each other.

Furthermore, the impact of the way that information is distributed may be modified by the form of the representation. Ainsworth & Peevers (2003) examined the interaction between the form of representations (tables, diagrams or text) and the number of representations (four simple representations or one complex one). Participants were provided with instructions about how to operate a complex device using these representations. Giving participants either one complex or four simple representations did not impact upon in the efficiency of participants' problem solving when using tables or diagrams. However, those participants given a single complex text spent much longer studying representations than those subjects who saw four texts but they were also more likely to find the ideal solution to the task.

It is apparent that whilst one of the advantages of using MERs is that information can be distributed to simplify individual representations, little research has directly addressed that question. Most of the experimentation holds informational equivalence across representations constant in order to explore computational non-equivalence. Furthermore, describing the information in a representation is problematic as information differs in how explicitly it is represented (e.g. Kirsh, 1991). One possibility is to base it on a theoretical description assuming a perfect information processor with unlimited resources and knowledge. However, people are not perfect information processors, and they differ in their background knowledge, skills, and cognitive capacities. More research is needed to explore these issues.

7.3. Form

This aspect of designing for effective learning has received the most attention and there is consistent evidence that differences in the form of representational systems strongly impacts upon learning processes and outcome. However, the majority of research has concentrated on modality and sensory aspects of representations. This focus has meant that other forms of representational system remained significantly under-explored (Reimann, 2003) and one role that DeFT can play is to draw attention to that fact. Furthermore, a number of studies have found contradictions in the apparent usefulness of specific combinations of representations. For example, whether simultaneous presentation of written and spoken text is beneficial (e.g. Mayer & Sims, 1994; Kalyuga, 2000). From a DeFT perspective, one key reason for these differences is that it ignores the fact that MERs play different pedagogical functions. If MERs are used to support different computational properties and one of the representations is not needed by the learner, then simultaneous presentation of both representations it is not likely to aid learning, given the complex cognitive tasks and processing required by MERs. However, this is not the case if the first representation is needed to constrain understanding of the second representation or if both representations are needed to encourage deeper understanding. One strong prediction of DeFT, which requires empirical validation, is that different designs principles will apply for different pedagogical functions (see section 8).

7.4. Sequence

There are a number of approaches to deciding upon a sequence of representations and they can be placed on a continuum ranging from domain-specific to domain-general. Some researchers start with an analysis of the properties of the domain to be taught in order to identify any representational consequences. Only in the absence of any particular constraints arising from this domain analysis are more general representational factors considered. Alternatively, a representational perspective can be taken which favours a domain-general approach.

An example of a domain-specific approach can be seen in MathsCar, a multi-representational system to teach introductory calculus (Kaput, 1994). Kaput argues that when teaching calculus, introducing integration before differentiation best supports understanding. Consequently, he proposes representations such as velocity-time graphs should be introduced before position-time graphs. At a mid point on the continuum lies the approach of Plötzner (1995), who analyses one-dimensional motion in classical physics problems to argue that qualitative knowledge should be taught before quantitative knowledge and consequently qualitative representations should be introduced before quantitative ones. Evidence for this proposal is provided by a cognitive model (SEPIA) and by the performance collaborating pairs taught with different sequences of qualitative and quantitative representations (Plötzner, Fehse, Kneser, & Spada, 1999).

At the other end of the continuum, a more domain-general approach can be seen in Kulhavy's model of text learning with organized spatial displays, which suggests that graphical representations should precede text. Verdi, Johnson, Stock, Kulhavy, & Ahern, (1997) showed that learners presented with a visual display before related text recalled significantly more information than when presented with text and then the visual display. Another domain-general is to introduce representations in such a way as to increase their abstraction. For example, COPPERS (Ainsworth *et al*, 1998) presents coin problems to children first as pictures but then through increasingly abstract representations such as mixed text and pictures and then text only and finally as algebra. Although, this approach can be seen in many systems, its validity has rarely been evaluated. It does seem reasonable to start by offering learners the least complex available representations. This may be the most concrete/least expressive representation that the increasing abstraction route suggests. However, in some situations, concrete and realistic representations can actually be more complex for learners (e.g. Lowe, 1999).

The question of whether learners make strategic decisions about when to change a representation or introduce a new one has been addressed by Cox (1996) who allows users to move at will between their self-created representations. He argues that this can be beneficial as it can help learners to resolve impasses. However, in line with Anzai (1991), Cox also found evidence to suggest that switching between representations can also be symptomatic of less understanding as learners tried to identify which representation helped them move nearer the goal without success. Another possibility is that learners should switch when they have exhausted all of the information available in the representation they are currently using. Graphs and Tracks (Trowbridge, 1989) exploits this technique to good effect. For example, help provided by the system suggests that users should switch from a velocity-time to a distance-time graph in order to gain information about the represented object's starting position.

Alternatively, the system may take responsibility for determining when to change the representation. In this case, the task for the system is to determine when users have learnt all they can about the domain with the given representations, but not switch so soon (or so often) that the learning demands of the new representations overburden the user. Unfortunately, as Resnick & Omanson (1987) observe, it is possible to introduce new representations too late. In their study of children learning to subtract using the standard written symbols, Dienes blocks were introduced to help children understand this task in a more conceptual way. The researchers were disappointed by how little children referred to the blocks and suggest that once children had reached automated performance with symbolic manipulation, it does not easily allow for application of principled knowledge. If this finding generalises to other domains, it suggests that a new representation should be introduced before learners have achieved automated performance with an existing representation. This raises the question of what aspects of learners' behaviour would need to be captured by a system and interpreted in a student model to be able to switch representations appropriately.

It is apparent that although there are many questions about sequencing representations, few of them have comprehensive answers. Deciding on a specific order of representations will almost always require a domain analysis, which could be supplemented with general representational principles. However, deciding about when to switch representations has received little attention and awaits systematic experimentation.

7.5. Translation

The above design parameters apply to non-computational media, but only computers can automatically support translation between representations and have many more way to indicate the connection between representations. Computer environments differ in how actively they support learning and whether this support is provided at the syntactic level or the semantic level.

The least active way that environments (computational or not) provide support for relating representations is in the use of implicit cues. For example, the relation between representations is easier to identify if they have consistent labels. DuFour-Janvier, *et al* (1987) suggest that children have a tendency to recognise that two representations concern the same problem only when they contain the same numbers. Other cues include using the same colours to represent the same objects over different representations.

More active support is seen when learners can select part of a first representation and see how this corresponds to a second representation. For example, in Brüncken, Plass & Leutner, (2003) learners can click on a hyperlink and arrows point to equivalent part of an accompanying picture.

Sometimes, learners act on one representation and see the results of those actions in another. This is commonly referred to as dyna-linking. For example, graphical calculators present dyna-linked algebraic expressions and graphs. Dynamic linking of representations is assumed to reduce the cognitive load upon the student - as the computer performs translation activities, students are freed to concentrate upon their actions on representations and their consequences in other representations. Kaput (1992) argues this is particularly beneficial when the representations involved are expressing actions sequences rather than just final outcomes as previous research has shown just how difficult this task is for learners (e.g. Resnick & Omanson, 1987). However, direct empirical support for the benefits of dyna-linking is not easy to find. For example, van der Meij & de Jong (2003) found no difference in learning between separate or dyna-linked representations.

Finally, some systems require learners to actively integrate representations and monitors students success in so doing. Bodemer *et al* (2004) gave learners spatially separated pictorial and symbolic representations of stats concepts and asked them to drag the symbols and dropping them within the pictures. Compared to split source or integrated representations, learners did better when required to integrate representations.

Only a few systems can vary the amount of help for relating representations that they offer to learners. However, this is probably the ideal. For example, there are reasons to hesitate about the invariable dynamically linking of representations. If the aim of instruction is to encourage users to understand the mapping between representations, then we may be in danger of over-automating the process. This over-automation may not encourage users to actively reflect upon the nature of the connection and could in turn lead learners to fail to construct the required deep understanding. Alternatively, this may encourage learners to attempt to link concepts that are currently beyond their level of understanding. Van Labeke & Ainsworth (2001) developed an approach based on contingency theory (Wood, Bruner, & Ross, 1976) in their simulation environment. Contingent translation proposes that the support provided to learners for any given task should vary depending upon their performance. As a learner succeeds, support should be faded out, but upon failure, then the learner should receive help immediately. Seufert (1999) provides support for varying the amount of support according to a learner's expertise. She found that only learners with a certain amount of prior knowledge benefited from help with translation between representations. High prior knowledge learners did not benefit as presumably they could make these links for themselves. Low prior knowledge students also did not benefit because they became overwhelmed. It was only learners with an intermediate level of prior knowledge who benefited from this help.

The level at which help is provided has received less direct research. Seufert (2003) provided help at a deeper level whereas most other research (particular with computers) has used more surface strategies. Van-Labeke & Ainsworth (2003) report a case study of three learners working with complex multi-representational software for up to eight hours and found they differed in their strategy for relating representations. One learner without relevant background knowledge used dyna-linking to support his surface level strategy for relating representations, whereas the two learners with more background knowledge used dynalinking less and attempted to relate representations using deeper structural features. Consequently, it may be the case that both the degree of support and the level at which this help is provided should vary depending on learners' expertise.

8. Design Heuristics

One key roles of DeFT is that it helps make clear the complex demands faced by learners when interacting with MERs. It adds to the substantial literature on cognitive load accounts of learning with MERs (e.g. Kirschner, 2002; Kalyuga, Chandler, & Sweller, 1999; Mayer & Moreno, 2002; Sweller, 1988) by identifying factors that add to cognitive load in these situations. Thus, it can help explain why sometimes learning with MERs is successful and sometimes not by examining which cognitive tasks learners mastered and which they did not. In addition, by emphasising the pedagogical functions of MERs and their implications for the cognitive tasks associated with MERs, DeFT provides an alternative way to consider how multi-representational systems might be designed to support learning. These proposals should be read as heuristics for guiding experimentation not as cast in stone principles.

When MERs are used to support complementary functions, learners need to understand each representation in isolation, how to select appropriate representations but need not understand the relation between them. The main design considerations therefore become one of selecting appropriate representations for the situation and the learners, rather than supporting learners in mastering the complex task of relating representations. Consequently, systems could provide dyna-linked representations and/or minimise co-presence of representations as learners often attempt to translate between co-present representations even if they do not need to do so to achieve the task.

When MERs are used to constrain interpretation it is imperative that the learner understands the constraining representation. Thus, using concrete representations with simple format and operators is ideal. But, in addition and in contrast to the first use of MERs, designers need also to ensure that learners understand how the constraining representation relates to the constrained representation. Consequently, designers must find ways of signalling the mapping between representations without again over-burdening learners by making translation complex. DeFT predicts that these representations should be co-present. Factors that increase the perceived similarity between representations such as similarity in labelling should be applied, as could dynalinking where appropriate.

The third function is when MERs are designed to allow learners to construct a deeper understanding of a domain. This goal provides designers with hard choices. If users fail to translate across representations, then abstraction and extension cannot occur. Learners find it difficult to translate over representations that are superficially dissimilar, but if made too easy, for example, by providing representations that do not provide sufficiently different views on a domain, then abstraction of invariances does not occur. However, if the system performs all the translation activities for students by dyna-linking the representations, then students are not afforded the opportunity to actively construct this knowledge for themselves. Approaches such as those of Bodemer *et al* (2004) and Van-Labeke & Ainsworth's (2002) may provide one solution to this issue. Although little research has addressed the way that differences in the way that information is distributed influence translation, Ainsworth *et al*, (1997) found tentative evidence that increasing the redundancy of information between (fairly simple) representations increased learners' abilities to reconcile representations that differ in format. In addition, to maximise opportunities for learners to build cognitive links over representations, then representations should be co-present. Although, much is still left to uncover, researchers are beginning to specify the factors that encourage or discourage deeper understanding.

9. Conclusion

This paper has illustrated the DeFT framework that describes some of the important aspects of learning with MERs. It clarifies the pedagogical functions that MERs serve, the often-complex learning demands that are associated with their use and in so doing aims to consider the ways that different designs of multi-representational systems impact upon the process of learning. It is hoped that DeFT will prove to be helpful to other researchers analysing learning with MERs by highlighting areas of study that are relatively under-investigated, provide an explanation for the apparently opposing findings, offer a common language for describing aspects of system design allowing generalisations across studies to be more easily achieved and ultimately aid the development of design heuristics and principles for learning with more than one representation.

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